Title: Drawing the Line on Diffusion

Brief Overview:

    The students will perform CBL activities using the conductivity and temperature probes to measure the rates of diffusion at different temperatures. The data from these activities will be used to determine the best fit line for diffusion as a function of time. Students will determine the domain and the range of the functions and compare the rates of change from the graphic, symbolic, and numeric representations. (Note: This unit is designed as an interdisciplinary activity with day one conducted in biology and day two for Algebra I/II.)

Links to NCTM Standards:

- **Mathematics as Problem Solving**
  Students will use the TI-82/83 and the CBL to collect, interpret, analyze, and draw conclusions from data and graphs. The students will apply mathematical modeling to real-world problem situations.

- **Mathematics as Communication**
  Students will use appropriate mathematical and scientific language to formulate equations, express generalizations, and make predictions discovered through investigations. They will ask clarifying and extending questions about linear functions and rate of change.

- **Mathematics as Reasoning**
  Students will make and test conjectures by determining the best fit lines to represent their data. They will compare the rate of change of these functions and construct simple, valid arguments to support their conclusions.

- **Mathematical Connections**
  Students will recognize the equivalent numeric, symbolic, and graphic representations of slope of a linear equation. Students will understand the connection of slope with the process of diffusion in science.

- **Algebra**
  Students will use the graphs from stat plot to predict the best model for the data. Students will use data points from the diffusion activity to find the slope and equation of the linear function. Students will use the data collected in the lists of their TI-82/83 to discover the linear regression equation for the diffusion activity.

- **Functions**
  Students will represent the rate of diffusion as a function of the temperature of a solution. Students will represent and analyze this function as a linear model by using the numeric (data lists), symbolic (equations generated algebraically and by calculator regression), and graphic (stat plots) forms.

- **Geometry from a Synthetic Perspective**
  Students represent the problem situation involving constant rate of change with the geometric model of a line.

- **Geometry from an Algebraic Perspective**
  Students will use the coordinates of two points on the line (two data points) to deduce the slope of the line which is a property of this geometric figure.

- **Statistics**
  Students will draw inferences from tables and graphs that summarize data from real-world situations and use curve fitting to predict to predict from data. This data analysis will be based on the diffusion experiment using the CBL. Students will interpret and communicate the outcomes of the experiment using an interdisciplinary approach to connect math and science outcomes.

Links to Maryland High School Core Learning Goals:

Skills for Success

- **1.2.5**
  The student will evaluate learning experiences.

- **2.2.2**
  The student will evaluate relevance and utility of information for specific purposes.

- **4.2.2**
  The student will use appropriate technologies to access, store, analyze, and communicate information.

- **5.2.4**
  The student will demonstrate understanding of and assume various roles in groups.

Science

- **1.2.2**
  The student will test a working hypothesis.

- **1.3.2**
  The student will demonstrate safe handling of the chemicals and materials of science.

- **1.4.1**
  The student will use analyzed data to evaluate a hypothesis.

- **1.4.2**
  The student will compare data for two groups by representing their distribution graphically.

- **1.4.3**
  The student will determine the relationships between quantities and develop the mathematical model that describes these relationships.

- **1.4.6**
  The student will use spreadsheet, graphing, and database programs and probeware on computers and graphing calculators.

- **1.5.4**
  The student will use computers and/or graphing calculators to produce tables, graphs and spreadsheet calculations.

- **3.1.1**
  The student will be able to describe the unique characteristics of chemical compounds and macromolecules utilized by living systems.

Mathematics

- **1.1.1**
  The student will recognize, describe, and extend patterns and functional relationships that are expressed numerically, algebraically, and geometrically.

- **1.1.2**
  The student will represent patterns and functional relationships in a table, as a graph, and/or by mathematical expression.

- **1.2.1**
  The student will determine the equation for a line, solve linear equations, and describe the solutions using numbers, symbols, and graphs.

- **2.1.1**
  The student will describe the characteristics of geometric figures and will construct or draw geometric figures using the coordinate plane.

- **2.1.2**
  The student will identify and verify properties of geometric figures using concepts from algebra and using the coordinate plane.

- **3.1.1**
  The student will design and/or conduct an investigation that uses statistical methods to analyze data and communicate results.

- **3.2.1**
  The student will make informed decisions and predictions based upon the results of simulations and data from research.

Grade/Level:

Grades 9 - 12; All levels of Biology, Algebra I, Algebra II

Duration/Length:

Two to three class periods (variable)

Prerequisite Knowledge:

Students should have working knowledge of the following:

- Concepts in diffusion and osmosis
- Use of the CBL and TI-82/83 graphing calculator
- Linear functions and their characteristics
- Numeric, symbolic and graphic representations of slope

Objectives:

Students will be able to:

- conclude that the membrane of the dialysis tubing is semi-permeable by collecting data and drawing conclusions.
- conduct an activity to test the effect of temperature on the rate of diffusion.

- recognize the relationship between temperature and the rate of diffusion by collecting experimental evidence.
- determine the domain and range of a linear function from the numeric and graphic representations of collected data.
- determine the linear function for collected data.
- compare rates of change of linear functions using real-world examples.

Materials/Resources/Printed Materials:

- Table salt (NaCl)
- TI-GRAPH LINK
- Balance
- Student worksheet - conductivity experiment
- Student worksheet - linear functions/rate of change
- Assessments for biology and algebra

Per Student Group:

- CBL system
- TI-82/83 graphing calculator with unit-to-unit link cable
- Conductivity probe
- Conductivity standard - sodium chloride solution (1000uS/cm conductivity)
- Temperature probe
- Dialysis tubing (three 15 cm lengths per group)
- 2 L distilled water
- 3 800 mL beakers per group
- 1 15 mL graduated cylinder
- Pipette
- 100 mL 5% NaCl solution (95 mL distilled water and 5 gm NaCl)
- 6 pieces twine
- Ice bath

Development/Procedures:

Before Day 1:

1. Teacher could prepare 5% NaCl solution.

2. Teacher should cool some distilled water.

3. The Meridian program, "Diffuse2", must be loaded onto each TI-82/83.

4. Hand out student worksheets, and assign pre-lab section for homework.

Day 1:

1. Review pre-lab answers and diffusion information.

2. Place students in groups.

3. Distribute materials.

4. Run experiment following student procedure sheet.

5. Save all data on calculators. Directions can be found on student procedure sheet.

6. Assign any incomplete lab questions for homework.

Day 2:

1. Recall data from Conductivity Experiment. (Students who did not participate in the lab activity can receive data using LINK on the TI-82/83.)

2. Place data in lists: cold water (L1, L2), room temperature (L3, L4) and hot water (L5, L6).

3. Set STAT PLOTS: PLOT 1 (L1, L2), PLOT 2 (L3, L4) and PLOT 3 (L5, L6).

4. Examine graphs of PLOT 1, 2, 3 with the students to review slope/rate of change.

5. Complete student activity sheet for day two.

6. Discuss activity sheet with emphasis on math/science real-world application of linear functions.

Extension/Follow Up:

Now the students can do a lab to investigate the effect solute concentration has on the rate of diffusion, and mathematically compare the findings to the relationship of temperature on diffusion.

References:

Explorations in Biology for the TI-82 and TI-83. Meridian Creative Group, 1996

Authors:

Richard M. Bohonek
North County High School
Anne Arundel County, MD

Jennifer M. Lombardi
North County High School
Anne Arundel County, MD

Patricia A. Kuehne
North County High School
Anne Arundel County, MD

Stacie L. Moon
North County High School
Anne Arundel County, MD
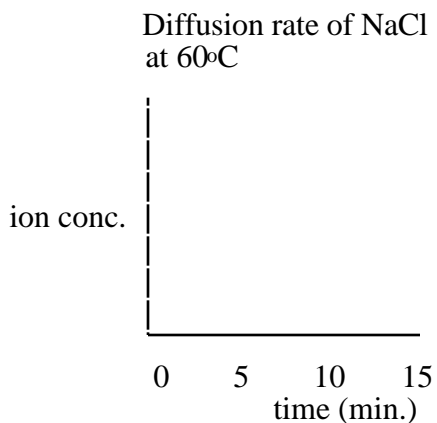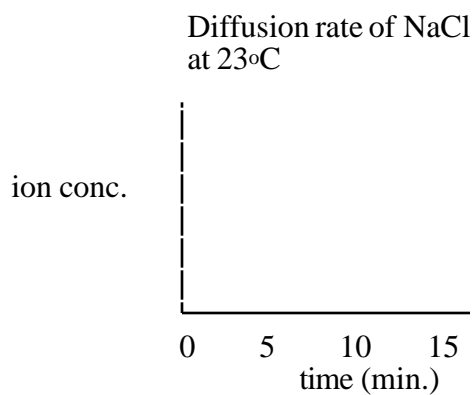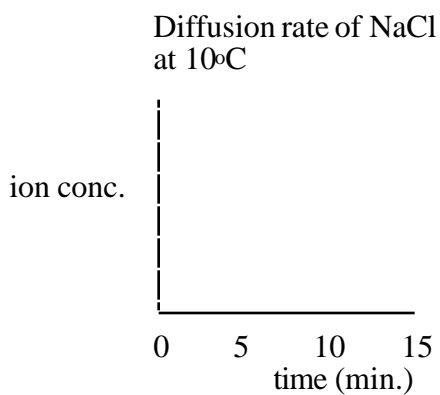
Student Procedure Sheet

1. Put 400 ml of distilled H_2O into each of the 800 ml beakers.

2. Place one beaker on a hot plate, and heat to around 60º.

3. Place the second beaker in an ice bath to cool to around 10º.

4. Allow the third beaker to reach room temperature.

5. Each group of students will cut dialysis tubing into three 15 cm lengths.

6. Tie off one end of the tubing with twine. Do not pull the twine too tight or the tubing will rip.

7. Fill each tube with 15 ml of the 5% NaCl solution.

8. Tie off other end of the tubing, making sure that there is no air in the tube.

9. Take the temperature of the first beaker to be tested using the temperature probe.
    a. plug temperature probe into channel one in the CBL.
    b. turn on the CBL
    c. press mode
    d. when temperature stabilizes, record the temperature

10. Connect CBL to TI-82/83

11. Turn on TI-82/83 and press program (PRGM)

12. Select program to Diffuse2

13. Follow the directions on the TI-82/83 to begin the conductivity test on one of the beakers.

14. Complete the same test on each of the beakers. The TI-82/83 requires the calibration of the probe between each test.

15. After each test, save the data collected in the TI-82/83 to be analyzed later.
    a. press **PRGM > >** to highlight **NEW**
    b. press **ENTER**. The TI-82 is in alpha mode. Type the name of the program. Press **ENTER**.
    c. press **2nd [RCL]**. Rcl appears at the bottom of the window. Type list to be recalled; for example, press **2nd [L2]**. Press **ENTER, STO>, 2nd, L2, ENTER.**
    d. Repeat step C for next list to be stored. For this example, L3 is used.
    e. stop

16. During the experiment, complete worksheet that accompanies the activity.

Conductivity vs. Temperature

Pre-Lab Questions
1. Diffusion is the movement of molecules from one area of high concentration to an area of low concentration. Predict what would occur if a 5% NaCl (sodium chloride) solution were added to distilled water at the following three temperatures.

Diffusion rate of NaCl
at 10°C

ion conc.

    0   5   10   15
         time (min.)

Diffusion rate of NaCl
at 23°C

ion conc.

    0   5   10   15
         time (min.)

Diffusion rate of NaCl
at 60°C

ion conc.

    0   5   10   15
         time (min.)

2.   Hypothesize why there would be a difference in the rate of diffusion at each temperature.

_____

_____
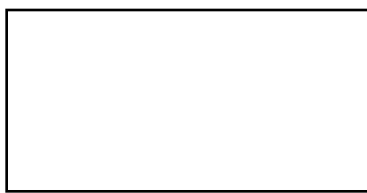
_____

Conductivity vs. Temperature

Lab Questions

1. Sketch the graph as it appears on your TI-82/83. Label all parts of the graph.
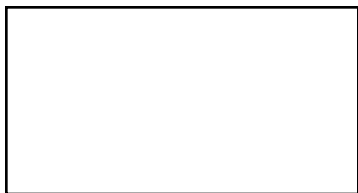
Temperature _____   Temperature _____

Temperature _____

2. Record your data on the following table. (C = conductivity)

| Temperature	Ci	C 1min.	C 3mins.	C 5mins.	C 6mins.	C 9mins.	C 15mins.	Δ C

3. Why did the conductivity of the distilled water change during the experiment?
_____
_____
_____

4. Estimate the rate of diffusion by finding the slope of a line between two points on each "diffusion" graph. For example, consider the diffusion graph shown below. To estimate the rate of diffusion, find the slope of the line between points (a,b) and (c,d). The formula for the slope of a line is: Slope= $\frac{d-b}{c-a}$

a. graph 1 temperature _____
   slope =___-___ = _____
            -
b. graph 2 temperature _____
   slope =___-___ = _____
            -
c. graph 3 temperature _____
   slope =___-___ = _____
            -
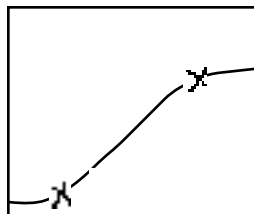
5. Did your "system" reach equilibrium? Explain your reasoning.
_____
_____
_____

6. Some animals are cold-blooded, which means that their body temperature is the same as their surroundings. If the temperature of the animal's surroundings decreases, describe how this may affect the animal's cellular activities.
_____
_____
_____
_____
_____

7. Basing your decision on the results of this experiment, describe several advantages of warm-blooded organisms.
_____
_____
_____
_____
_____
_____

8. In some cases, a person submerged in cold water for a prolonged period of time suffers less brain damage than expected. Describe how this may occur.
_____
_____
_____
_____
_____

9. Extension:
Now that you understand the effect of temperature on the rate of diffusion, what kind of effect does solute concentration have on rate of diffusion? Use the definition of diffusion in your explanation.
_____
_____
_____
_____
_____

Diffusion as a Linear Function

1. (A) Sketch graphs of diffusion as a function of time from the data in LIST 1-6 on your
    TI-82/83. Label the independent and dependent axis.

| Cold Water | Room Temperature | Hot Water |
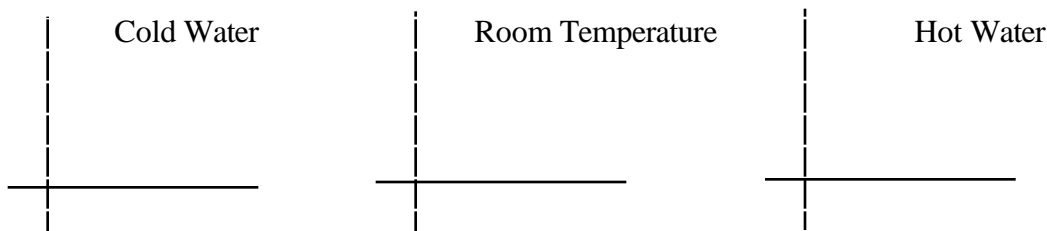
   (B) Which graph has the greatest rate of change? _____ What does this imply about
        the relationship between rate of diffusion and water temperature? _____
        _____

   (C) Do the graphs represent increasing or decreasing functions? _____
        Explain: _____

2. (A) Using the data in LIST 1-6, calculate linear regression equations for each experiment.
    (Approximate to nearest hundredth.)

    COLD WATER: Y1 = _____

    ROOM TEMP: Y2 = _____

    HOT WATER: Y3 = _____

   (B) Compare the slopes of the linear regression equations. Which equation contains the
        greatest slope? _____ Is this consistent with your graphs in #1? _____

3. (A) Using the data in LIST 1-6, use the initial and final data points from each activity to
    compute the rate of change (slope) algebraically. (Approximate data points to nearest
    hundredth.) Remember, slope = y/ x.

	L1	L2	L3	L4	L5	L6
INITIAL						
TERMINAL						

        SLOPE: Y1_____   Y2_____   Y3_____

   (B) Which slope is the greatest? _____

   (C) How does this slope compare with the slope of the corresponding regression equation in
        #2 ? _____

4. (A) Using the graphic and numeric representations of the data, define the domain and range for each function in #3. (Approximate to nearest hundredth.)

Y1: Domain _____   Range _____

Y2: Domain _____   Range _____

Y3: Domain _____   Range _____

   (B) Time is the _____ variable and diffusion
        is the _____ variable.

   (C) The time(domain) is set by the CBL activity for this experiment. Data was collected for 15 minutes in each solution. The highest level of diffusion attained during this amount of time in any solution was _____. This maximum diffusion occurred in which temperature solution? _____ Explain how this is consistent with the graphs in #1 and the equations in #2. _____

5. If the CBL activity had collected data for an extended period of time, the solutions would have reached equilibrium. What would be the rate of change at that point? _____ Sketch one of the graphs from #1 to show the behavior of the function including this point at this time and times beyond.

6. Juan used powder detergent to do a load of laundry. His jeans and other dark articles came out with a light flaky residue on them. What suggestion would you offer to Juan to eliminate this problem?
_____

Performance Assessment

Teacher's Guide

Introduction

In this performance assessment, the students will demonstrate mastery of the objectives for the learning unit, "Drawing the Line on Diffusion".

Objectives Covered

The students will be able to:

- conclude that the membrane of the dialysis tubing is semi-permeable by collecting data and drawing conclusions.
- conduct an activity to test the effect of temperature on the rate of diffusion.
- recognize the relationship between temperature and the rate of diffusion by collecting experimental evidence.
- determine the domain and range of a linear function from the numeric and graphic representations of collected data.
- determine the linear function for collected data.
- compare rates of change of linear functions using real-world examples.

Tools/Materials Needed for Assessment

- Student assessment sheet
- Pencil
- Calculator (optional)

Administering the Assessment

This assessment can be administered together either in the student's biology or algebra class, or it can be divided into the appropriate sections. The scores can then be shared with the two teachers, or divided into sections.
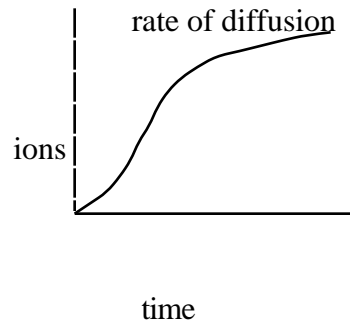
Performance Assessment

Student Response Sheet

BIOLOGY

Directions: Read the statement and select the best response.

1. As a result of diffusion, the concentration of many types of substances:
    A. remains greater inside a membrane
    B. becomes balanced on both sides of a membrane
    C. remains greater on the outside of a membrane
    D. becomes imbalanced on both sides of a membrane

2. Diffusion : to a lesser concentration
    A. osmosis : through a membrane
    B. active transport : passive transport
    C. equilibrium : to a greater concentration
    D. solvent : to a solute
    E. solute : to a solvent

3. What is the relationship of temperature to rate of diffusion?
    A. inversely proportional
    B. jointly proportional
    C. directly proportional
    D. indirectly proportional

4. Diffusion takes place
    A. through a lipid bilayer membrane
    B. from an area of low concentration to an area of high concentration
    C. only in liquids
    D. from an area of high concentration to an area of low concentration

5. Think about the experiment you conducted with the rates of change of diffusion. If you were to spill a bottle of perfume somewhere in your house, in which room would the molecules of scent reach equilibrium first?
    A. a 20^0 C cool basement
    B. a 25^0 C a warm kitchen
    C. a 23^0 C room temperature living room
    D. a 28^0 C hot, steamy bathroom

6. Which part of your body would be most similar to the semi-permeable membrane?
    A. surface of a bone
    B. membrane of a red blood cell
    C. outer layer of skin
    D. covering of the eye

Directions: Using two to three complete sentences, respond to the following questions:

rate of diffusion

ions

time

7. In the above graph, what phenomena is occurring as the slope approaches zero (levels out)?

_____
_____
_____

What occurs while the graph is increasing?

_____
_____
_____

Directions: In paragraph form, respond to the following statement:

8. Solute concentration is directly proportional to rate of diffusion. Propose a hypothesis about the relationship of solute concentration to rate of diffusion. Design an experiment to test your hypothesis using a CBL and TI-82/83.

_____
_____
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_____

Algebra I and II

Directions: Read the statement and select the best response.

1. A linear function is increasing when:

    A. as x increases, f(x) increases.
    B. as x decreases, f(x) increases.
    C. as x increases, f(x) decreases.
    D. as x decreases, f(x) decreases.

2. What two (2) items are necessary to determine the equation of a line?

    A. point and slope.
    B. x-coordinate and y-coordinate.
    C. two (2) x-coordinates.
    D. two (2) y-coordinates.

3. Write a mathematical formula for finding the slope of a line.

    _____

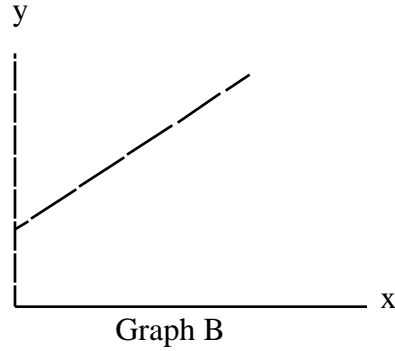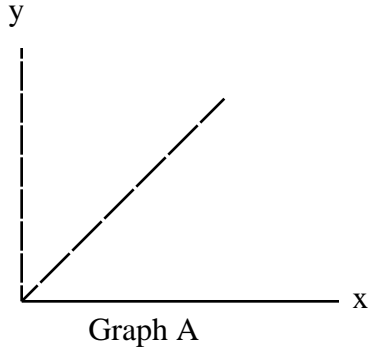4. Given the four graphs, respond to the following questions:

    Graph #1   Graph #2   Graph #3   Graph #4
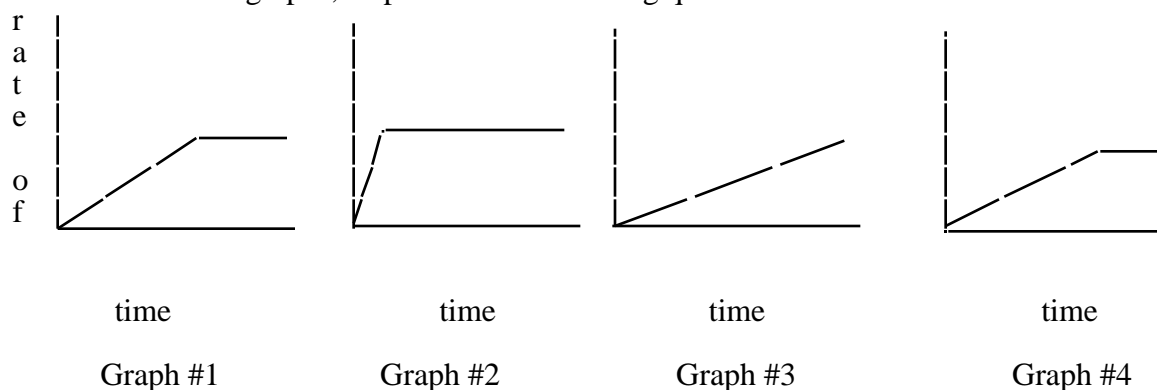
    A. Which graph represents the greatest rate of change?
    _____

    B. Which graph attains a zero (0) slope first?
    _____

    C. What does graph #3 suggest about the rate of change as compared to the other
       three graphs?
    _____

    D. What does the horizontal portion of the graph indicate about the rate of change?
    _____

y   y

Graph A   Graph B

Directions: Given the following graphs, respond to the questions below:

5. If a regression equation for Graph A is **y=x**, write a possible equation for Graph B.

          _____

6. Given the data table find the slope of the line.

x	y
4	11
6	14

Directions: Using two to three complete sentences, respond to the following question:

7. Natasha decided to make some sweetened tea. She would like her tea ready to drink as soon as possible. Based on the conductivity experiment you have completed, how would the temperature of the tea affect the time it takes the sweetener to dissolve?

_____
_____
_____
_____

8. Explain, in paragraph form, how to determine the slope of a line given only its graph?

_____
_____
_____
_____
_____
_____
_____
_____
_____
_____

Performance Assessment

Scoring Guide

BIOLOGY

Directions: Read the statement and select the best response. Each response is valued at 1 point.

1. As a result of diffusion, the concentration of many types of substances:
   A. remains greater inside a membrane
   B. *becomes balanced on both sides of a membrane*
   C. remains greater on the outside of a membrane
   D. becomes imbalanced on both sides of a membrane

2. Diffusion : to a lesser concentration ::
   A. *osmosis : through a membrane*
   B. active transport : passive transport
   C. equilibrium : to a greater concentration
   D. solvent : to a solute
   E. solute : to a solvent

3. What is the relationship of temperature to rate of diffusion?
   A. inversely proportional
   B. jointly proportional
   C. *directly proportional*
   D. indirectly proportional

4. Diffusion takes place
   A. through a lipid bilayer membrane
   B. from an area of low concentration to an area of high concentration
   C. in liquids
   D. *from an area of high concentration to an area of low concentration*

5. Think about the experiment you conducted with the rates of change of diffusion. If you were to spill a bottle of perfume somewhere in your house, in which room would the molecules of scent reach equilibrium first?
   A. a 20^0 C cool basement
   B. a 25^0 C a warm kitchen
   C. a 23^0 C room temperature living room
   D. *a 28^0 C hot, steamy bathroom*

6. Which part of your body would be most similar to the semi-permeable membrane?
   A. surface of a bone
   B. *membrane of a red blood cell*
   C. outer layer of skin
   D. covering of the eye

Directions: Using two to three complete sentences, respond to the following questions. Refer to the graph on student worksheet.

7. In the above graph, what phenomenon is occurring as the slope approaches zero (levels out)?

    1pt   ***The ions are reaching equilibrium.***

What occurs while the graph is increasing?

    1pt   ***The ions are diffusing across the membrane.***

Directions: In paragraph form, respond to the following statement:

8. Solute concentration is directly proportional to rate of diffusion. Propose a hypothesis about the relationship of solute concentration to rate of diffusion. Design an experiment to test your hypothesis using a CBL and TI-82/83.

    4 = Student includes:
        a detailed hypothesis, including why they believe that
        step-by-step procedure to test hypothesis, including all materials
        experiment includes use of CBL and TI-82/83

    3 = Student includes:
        a hypothesis
        basic procedure to test hypothesis with a list of some materials
        experiment includes the use of CBL and TI-82/83

    2 = Student includes:
        a hypothesis
        procedure to test hypothesis, no materials or explanations
        experiment includes either the CBL or TI-82/83

    1 = Student includes:
        hypothesis or procedure attempted

    0 = Blank paper

Algebra I and II Scoring Guide

Directions: Read the statement and select the best response. Each response is valued at 1 point.
1. A linear function is increasing when:

       A. as x increases, f(x) increases.
       B. as x decreases, f(x) increases.
       C. as x increases, f(x) decreases.
       D. as x decreases, f(x) decreases.
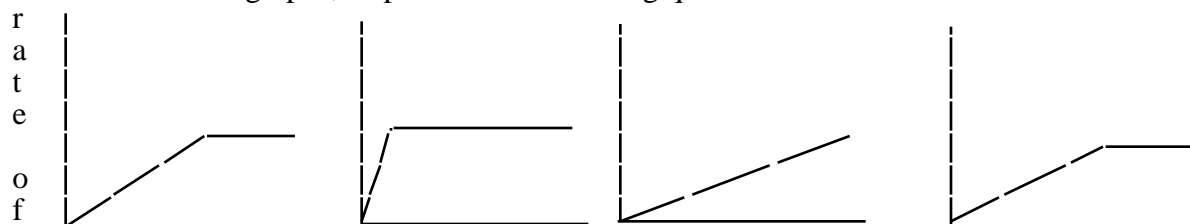
2. What two (2) items are necessary to determine the equation of a line?

       A. point and slope.
       B. x-coordinate and y-coordinate.
       C. two (2) x-coordinates.
       D. two (2) y-coordinates.

3. Write a mathematical formula for finding the slope of a line.
      $m = y/x$, $m = y_2 - y_1 / x_2 - x_1$, slope = rise/run, slope = change in y / change in x

4. Given the four graphs, respond to the following questions:

    time   time   time   time

   Graph #1   Graph #2   Graph #3   Graph #4

    A. Which graph represents the greatest rate of change?
       graph #2

    B. Which graph attains a zero (0) slope first?
       graph #2

    C. What does graph #3 suggest about the rate of change as compared to the other three graphs?
       the rate of change has not leveled off (or not reached equilibrium)

    D. What does the horizontal portion of the graph indicate about the rate of change?
       rate of change equals 0

Directions: Given the following graphs, respond to the question below:

y   y

x   x
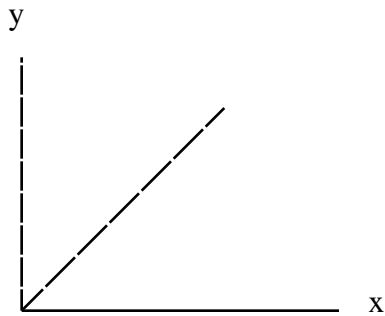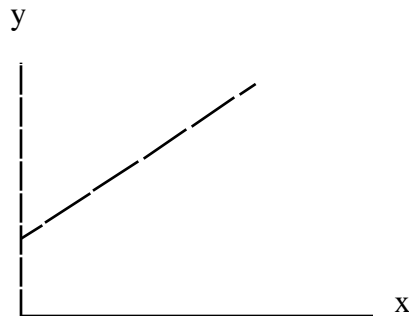
Graph A   Graph B

5. If a regression equation for Graph A is **y=x**, write a possible equation for Graph B.
   (2pts)   *y = .5x + 2*   (sample answer)
   2pts = student includes slope with 0<m<1 and y intercept with b>0
   1pt = student includes either correct slope or correct y intercept
   0pt = blank paper

6. Given the data table, find the slope of the line.

x	y
4	11
6	14

   (1pt)   *m = 3/2*

   Directions: Using two to three complete sentences, respond to the following question:

7. Natasha decided to make some sweetened tea. She would like her tea ready to drink as
    soon as possible. Based on the conductivity experiment you have completed, how would
    the temperature of the tea affect the time it takes the sweetener to dissolve?

    (1pt)   *sugar would dissolve or diffuse more quickly in hot water*

8. Explain, in paragraph form, how to determine the slope of a line given only its graph.

   3 pts = if student includes:
           A. answer in paragraph form
           B. determines coordinates of two points (selects two points from the graph)
           C. use the formula for slope, explains slope as change in y/ change in x, OR
               use point slope form with y intercept as one of the points

   2pts = if student includes:
           just B and C

   1pt = if student includes:
           just B OR C

   0pt = blank paper